Math 5376–001, Spring 2013
Constructing Rational Number and Operations

5–8 PM Tuesdays, 321 PKH

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Prerequisites: MATH 5375 (Constructing Whole Number and Operation)
Text materials: DMI's *Building a System of Tens* and *Making Meaning for Operations* casebooks (henceforth BST & MMO), and Empson & Levi's *Extending Children's Mathematics* (henceforth CGI). Additional materials will be provided in class or on the course web page.
Course home page: http://mathed.uta.edu/kribs/5376.html
Last day for withdrawal: March 29

Class policy on drops, withdrawals, academic honesty, and accommodating disabilities follows the University policy on these matters. Copies can be obtained upon request.

LEARNING OUTCOMES: After completing this course, students should be able to:

- identify and use appropriate representations of rational numbers in teaching situations, including conceptual, contextual, concrete, pictorial, and symbolic models
- analyze student thinking regarding rational numbers and elementary number theory
- design and implement research-based lessons to teach rational number concepts, including the development, selection and sequencing of problems
- justify, in simple terms, common algorithms for operating on rational numbers
- explain the correspondence between common fractions and decimals
- explain common divisibility tests
- use prime factorizations to determine the number of factors a given number has, or to find a number with a given number of factors

FORMAT: This course will study number and operation concepts in several ways: through work on challenging mathematical problems to develop our own mathematical abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the learning of rational number and operation concepts at the K–8 levels; and examining specific instances of K–8 students' representational and computational work, through both case studies and our own classroom practice.

Before class each week you will read articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class. We will typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. We will follow this up by discussing the assigned readings, as well as other topics related to problem solving. We will typically end class with time for reflection on how the topics we have discussed apply to our own classrooms.

During class discussions we will often refer back to work we have done earlier in the course, as well as in the companion course on whole numbers (WNO), so please bring your notes and papers from previous sessions to class.

POLICIES:

• Students who are not classroom teachers will need to make arrangements to interact with K–8 students for many of the assignments (those starred * on the calendar).

• Students are expected to be on time, prepared and ready to work every week. This class meets every Tuesday from 5 PM to 8 PM from January 15 to May 7, except for March 12 (spring break). Each student is allowed the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence.

• With the exception of student work samples, written assignments should be typed and use correct grammar and punctuation. (Diagrams, equations, etc. may of course be hand-drawn.)

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic submission during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include rough drafts sent for consultation.

• Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at Session 15. The new grade replaces the original. Students should consult with the instructor before submitting a revision.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop numerical concepts in your students. All of these are detailed in the next section.

Assignments

1. Journal

On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' mathematical work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); your preparation and participation grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *"What's a fraction?"*. Ask several students to define or explain what fractions are (use age-appropriate vocabulary, but be careful not to use suggestive terms like part-whole). Report their responses verbatim, and then compare them with the conceptual models for fractions identified in the readings. Do their definitions cover all the conceptual models?

J2 *Mini-interview.* Interview a single student about adding fractions. See web page for suggested prompts. This assignment should serve as a dry run for the student interview assignment, so see that portion of the syllabus for the general format, but keep in mind that this mini-interview should cover only these questions and thus be much smaller in scope.

J3 *Fraction representations.* Pose, to a group of K–8 students, the Classroom Challenge in the NCTM 2002 Yearbook, p. 105. You will want to give them enlarged copies of the figure(s) (but not the surrounding text!). Afterward, analyze each student's response as to whether it is one of the two common strategies identified in the text.

J4 *Mini-case study.* Pose, to a group of K–8 students, a question similar to those in readings we have discussed in class (another option is J4.5 below). Make sure it is appropriate to the students' grade level. Write about your question, what you expected, and what actually happened. Did anything surprise you? Please describe specific examples of what your students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of every student. Think of this as a dry run for the case study.

J4.5 *Fractions between* (for class discussion, not to write up). Pose, to a group of K–8 students, either Classroom Challenge in the 2002 NCTM Yearbook, pp. 257–258 (find three fractions between 7/11 and 7/12, or between 1/8 and 1/9). Present and analyze their responses. To what extent did they make use of complex fractions, as the children in the yearbook did?

J5 *Algorithms for dividing fractions.* The articles by Cramer et al., by Flores, Turner & Bachman and by Siebert present different approaches to making symbolic algorithms for division of fractions meaningful to students. In particular, they compare the "common denominator" algorithm to the traditional "invert and multiply" algorithm. Analyze and compare these two algorithms, first in terms of developing meaningful understanding (how can students come to understand each algorithm?), then in terms of computational efficiency, and finally overall.

J6 *"What's a decimal?".* Ask several students to define or explain decimals. Report their responses verbatim, and analyze them: identify what properties of decimals (and rational numbers in general) they identify, which they don't, and to what extent they show number sense.

J7 *Assessing rational number sense.* National and international assessments often show that even students who can compute fluently with fractions and decimals may have poor number sense (choosing the sum of the denominators, for instance, to estimate the sum of two fractions). Develop an assessment instrument that will allow you to analyze students' thinking and understanding with regard to number sense for rational numbers. Give the instrument and explain what your choices will allow you to learn. Pay particular attention to the specific numbers you use in the questions. (You are free to test it out on one or more actual students, but you need not report on that in the journal entry.)

J8 *One-problem paper.* As preparation for writing rigorously about math in the two-problem paper, write up a full explanation (with proof) of the solution to the following question from the Session 10 activity "Fractions and decimals in other bases": In what bases can the fraction 1/12 be written as a terminating (not repeating) "decimal"?

J8.5 *Proportional reasoning* (for class discussion, not to write up). Pose, to a group of K–8 students, a problem from either [NCTM], pp. 100-102, or Lamon Ch. 1 or 2. Analyze the thinking type and level in students' responses using the rubric on [NCTM] p. 101.

J9 *Text evaluation.* Evaluate the CGI text as a potential new text for this course, addressing (a) what new or surprising information it presented, (b) how useful you find it as a reference, (c) overall readability, and (d) if adopted, what should it replace?

J10 *Synthesis.* Looking back on the readings and discussions of the course to date, identify what are for you the major distinctions, (a) mathematically and (b) pedagogically, between fractions and decimals. Discuss explicitly your thinking about the role of units in each.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to assess her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a major topic from this course. Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and that this interview will not affect the student's grades; it will just help you understand how the student thinks. (Recording the interview will keep you from needing to make detailed notes during it.)

Before the interview, get a copy of recent written work by the student showing her/his ability to reason and problem-solve (the work need not be error-free, but there should be enough progress made to discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking him/her to explain the work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (see the handout on interviewing tips on the course web site). You will need to use both pre-prepared questions and ad hoc follow-up questions to develop a coherent line of questioning. Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error.* You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Begin with a brief introduction to provide context. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking. It must also center on a mathematical topic involving rational number and operation, or number theory.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then describe the relevant parts of the conversation in as much detail as you can manage. Include what you are thinking as you work with the students. Finish up by summarizing your evaluation of the students involved and saying what issues and questions you still have after this conversation. Include an analysis of the students' thinking, and questions the case raises for you. *It is important that your reflection address teaching issues beyond the one topic and set of students involved*, in order to document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

We will discuss the writing of cases in more detail before they are due, but you are encouraged to begin sooner if you have a good conversation fresh in your mind. I will be glad to help you.

4. 2-problem paper

In order to understand the concepts underlying rational number and operation (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. You *must* check the course web page or meet with me individually to approve and verify the problem you wish to write up.

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary.

Also select a single problem (*not* a lesson) from K–8 mathematics (possibly multi-part, and preferably from your own classroom) which you believe entails rational number and operation concepts similar to those involved in the college-level problem, and explain in detail what those concepts are, clarifying in a paragraph what common ideas the two problems share. Include the prompt in your paper. Limit your analysis to properties of the problems themselves; do not focus on student work (unlike in the other major papers).

I encourage you to show me a draft of your paper before final submission.

5. Lesson paper

In this course we will study the teaching and learning of ideas related to rational number and operation in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select a lesson which fosters learning these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on it. The lesson draft checkpoint includes items 1–3 below. The final lesson paper you submit must include *all* of the following components:

1. Select or develop a problem intended for use with the students you teach, which involves some aspect of rational number and operation or number theory. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept. Specify prerequisite knowledge.
2. Write a paragraph explaining what concepts from this course are entailed in this problem. (You may use deconstruction if it helps you identify them, but write in paragraph form.)
3. Add to the above written descriptions a short sketch of how you plan to use the problem in a lesson (including basic lesson structure), and meet with your instructor to discuss your progress. (This is the lesson draft checkpoint. The above items will also form part of your final paper.)
4. Write a lesson plan that uses the problem/activity as a significant problem-solving opportunity with your students. Include closure activities, and important discussion points.
5. Teach the lesson to your students (see me if this is problematic). Then write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying rational number concepts—or ability to apply them—changed as a result of the lesson. Be specific.
6. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include the problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan (select details!), and must be turned in with the main paper.
7. Give a brief (10-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the selected problem and lesson idea (not [necessarily] yet taught) is due at Session 9 (see step 3 above). Final documentation is due at Session 12, including a handout, with the presentations to be given at Session 16.

Bibliography

[FDRP] Carne Barnett, Donna Goldenstein, and Babette Jackson (Eds.) *Fractions, decimals, ratios, & percents*, Heinemann, Portsmouth NH, 1994. Available online (see link on class web page).

Kathleen Cramer *et al.*, Dividing fractions and problem solving, *MTMS* 15(6): 338–346, Feb. 2010.

[CGI] Susan B. Empson & Linda Levi, *Extending children's mathematics: fractions and decimals*, Portsmouth, NH: Heinemann, 2011.

[FSY] Alfinio Flores, Jeffrey Samson, and H. Bahadir Yanik, Quotient and measurement interpretations of rational numbers, *TCM[=Teaching Children Mathematics]* 13(1): 34–39, August 2006.

[FTB] Alfinio Flores, Erin E. Turner, and Renee C. Bachman, Posing problems to develop conceptual understanding: two teachers make sense of division of fractions, *TCM* 12(3): 117–121, October 2005.

Christopher M. Kribs-Zaleta, Oranges, posters, ribbons, and lemonade: concrete computational strategies for dividing fractions, *Mathematics Teaching in the Middle School* 13(8): 453-457, April 2008.

Susan J. Lamon. (1999). *Teaching Fractions and Ratios for Understanding: Essential Content Knowledge and Instructional Strategies for Teachers.* Mahwah, NJ: Lawrence Erlbaum Associates. [Available electronically through the UTA Library catalog at https://pulse.uta.edu]

[NCTM] Bonnie Litwiller and George Bright (Eds.), *Making sense of fractions, ratios, and proportions*, 2002 NCTM Yearbook, Reston, VA: NCTM, 2002.

Nancy K. Mack, Connecting to develop computational fluency with fractions, *TCM* 11(4): 226–232, 2004.

Enrique Ortiz, The roll out fractions game: comparing fractions, *TCM* 13(1): 56–62, August 2006.

Jack M. Ott, Daniel L. Snook, and Diana L. Gibson, Understanding partitive division of fractions, *Arithmetic Teacher* 39: 7–11, October 1991.

[BST] Deborah Schifter, Virginia Bastable, and Susan Jo Russell. (1999). *Building a System of Tens (Number and Operation, Part 1) Casebook.* Parsippany, NJ: Dale Seymour/Pearson.

[MMO] Deborah Schifter, Virginia Bastable, and Susan Jo Russell. (1999). *Making Meaning for Operations (Number and Operation, Part 2) Casebook.* Parsippany, NJ: Dale Seymour/Pearson.

Daniel Siebert, Connecting informal thinking and algorithms: the case of division of fractions, in [NCTM], pp. 247–256.

John P. Smith III, The development of students' knowledge of fractions and ratios, in [NCTM], pp. 3–17.

Susan B. Taber, Go ask Alice about multiplication of fractions, in [NCTM], pp. 61–71.

Calendar

A tentative schedule with topics is given below (subject to updating).

Sess.	Date	Rm.	Topic	Readings/Cases Due	HW Due
1	1/15	232	Representations of fraction	MMO8:1–4; CGI intro	—
2	1/22†	232	Fraction as part-whole and ratio	FDRP18,19; Ortiz; Smith; CGI1	J1*
3	1/29	230	Fraction as quotient	MMO4; FSY; CGI2	J2*
4	2/05	231	Adding & subtracting fractions	MMO5; Mack; CGI3	J3*
5	2/12	231	Multiplying frac'ns; frac. as operator	FDRP1,7; MMO6; Taber; CGI4	Interview*
6	2/19	235	Dividing fractions	MMO7:27; Kribs-Zaleta; Ott et al.	J4*
7	2/26†	231	Algorithms for dividing fractions	MMO7:28; Cramer; FTB; Siebert	J5
8	3/05	240	Repr'ns & place value for decimals	BST7:25–27; FDRP8; CGI5	Case study*
–	3/12		SPRING BREAK	some trashy novel	sleep in
9	3/19	235	Rel'nships btw. fractions & decimals	BST7:29, CGI6	J6*, lesson draft
10	3/26†	240	Adding & subtracting decimals	BST7:28; FDRP6,10; CGI7	J7
11	4/02	230	Multiplying & dividing decimals	FDRP14; CGI8	J8, 2PP drafts
12	4/09	235	Number systems and infinities	CGI9	Lesson paper*
13	4/16	232	Divisibility tests	2PP drafts	J9, draft feedback
14	4/23	232	Factors I	—	2-problem paper
15	4/30	232	Factors II	MMO8:5–7	J10
16	5/07	232	Final presentations	—	Presentations

† = TQ reflective prompt *after* class for TQ participants.
{BST/MMO/CGI/FDRP}n[: a–b] means Chapter n [sections or cases a through b only] of the given book.
See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).