Math 5379–001, Summer I 2016:
Measurement Concepts in K–8 Mathematics

May 12–June 2: Tue/Thu 5–8 PM, PKH487;
June 7–23: Tue/Wed/Thu 3:30–6:30 PM, PKH487

Instructor: Dr. Christopher Kribs
Office: 483 Pickard Hall
Phone: (817)272-5513, fax 272-5802
email: kribs@mathed.uta.edu
WWW: http://mathed.uta.edu/kribs/
Office Hours: before/after class & by appt.

Prerequisites: Graduate standing and consent of instructor
Text materials: DMI's *Measuring Space in One, Two, and Three Dimensions* casebook (see bibliography; henceforth MSP). Additional materials will be provided in class or on the course web page.
Course home page: http://mathed.uta.edu/kribs/5379.html
Last day for withdrawal: June 29

LEARNING OUTCOMES: The successful student will be able to:

- *explain relationships among measurable quantities*, including the origins of common formulas
- *identify common approaches & errors* in student learning of measurement concepts
- *use concepts of unit to facilitate mathematical learning*
- *describe and communicate* mathematical ideas
- *understand and analyze* the mathematical thinking of others (especially students)
- appropriate *ways to assess* understanding of measurement

FORMAT: This course will study measurement concepts in several ways: through work on challenging mathematical problems to develop our own measurement abilities (and communicating the results, to develop our expository abilities); discussing what research has discovered about the development of measurement abilities at the K–8 levels; and examining specific instances of K–8 students' measurement work, through both case studies and our own classroom practice.

Before class each week you will read articles and/or case studies from K–8 mathematics, and make notes on them in preparation for class discussions. You will also often work on mathematics problems outside of class, to facilitate their discussion in class (especially important if you often do not complete them in class before we discuss them in large group). We typically begin class by working on new mathematics problems and discussing their solutions, in both small and large groups. (If you have scissors, ruler, protractor, please bring them.) We follow this up by discussing the assigned readings, as well as other topics related to problem solving. We typically end class with time for reflection on how the topics we have discussed apply to our own classrooms. During class discussions we often refer back to work we have done earlier in the course, so please bring your notes and papers from previous sessions to class.

POLICIES:

- Students who are not classroom teachers will need to make arrangements (I can help) to interact with K–8 students for many of the assignments (those starred * on the calendar).
- Students are expected to be on time, prepared and ready to work every week. This class meets on each of the days listed on page 1. Each student is allowed the equivalent of one week's absence (3 total hours) for whatever reason without penalty. All subsequent absences (including arriving significantly late) will result in the reduction of the final course grade by one-half letter grade (5%) for each absence (even if you tell me ahead of time). *Students who miss an entire class must talk with the instructor prior to the next class.* See DMI handout for policy on making up work from a missed class.

• With the exception of diagrams and examples of student work, written assignments are expected to be typed and use correct grammar and punctuation. No cover pages necessary.

• Each student is allowed one late submission during the semester. The paper must be submitted before the beginning of the class period following that in which it was due. Papers not submitted by the end of class time on the due date are considered late. Submission of a late paper constitutes the student's agreement that this is the one allowed late assignment.

• Each student is allowed one electronic submission (in PDF) during the semester. Electronic submissions must be complete and not missing any ancillary materials such as student work necessary for grading. (If the electronic submission is made late, then it is both the only late paper allowed and the only electronic submission allowed.) This does not include drafts sent for consultation.

• Each student is allowed to submit one revised paper for a regrade, under the following terms: The revised paper and the graded original must be turned in together at Session 15. The new grade replaces the original. Students should consult with the instructor prior to submitting a revised paper.

• As a sign of respect for your peers and our common work, please keep all cellular phones, computers, and other electronic devices turned off during class. In emergencies cell phones may be set to vibrate only, and brief calls taken in the hallway outside.

• This course follows University policies on topics such as drops, withdrawals, academic integrity, accommodating disabilities, etc. Please see the attached supplement for further details.

GRADES: Your grade for the course will be determined by five elements, each of which has equal weight: (1) journal entries and participation, (2) a written student interview, (3) a short case study, (4) a paper detailing your own mathematical work, and (5) a lesson involving a problem you select to develop measurement concepts in your students. All of these are detailed in the next section.

Assignments

1. Journal

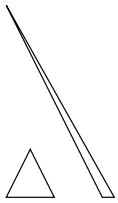
On days in which there is not a major assignment due, you will write a short (about one page) reflection in response to a prompt given below. Many of the prompts are also given in the DMI handouts distributed in class. Some involve "action research" reports in which you will write about your own students' measurement work. You will often use and discuss your responses in class, within your small groups and in large group. These reflections are to be turned in at the end of class; I will respond to them in writing and return them at our next class meeting. Grading will be limited to verifying that responses are appropriate in topic and scope (length).

Your journal entries will serve to document your preparation for class each day (and your growth over time); this component of your grade will be based half on your journal (entries should be complete each day before class) and half on your participation in class discussions (I expect participation in large-group discussion at least ten of the fifteen times we will meet).

J1 *Key ideas.* Write about what you see as the key ideas in measurement. See DMI handout for full prompt. Do *not* just list the measurement TEKS for the grade(s) you teach—reflect and identify major themes and issues (conceptual or practical).

J2 *Benchmarks mini-interview.* Interview a K–8 student about mental benchmarks for common measurement units. Ask the student to name, in each case below, something that is about one (a) inch long, (b) yard long, (c) mile long, (d) centimeter long, (e) meter long, (f) kilometer long. See interviewing tips handout for how to approach this (think of it as a dry run for the interview assignment, though the latter may address a different topic). Include an analysis.

J3 *Area reflection.* (a) Is the definition of "area" in your dictionary good enough to explain the meaning of the term to someone who had never heard the concept before? If not, how does it fail? Imagine you have a student with the learning disability "uni-dimensia", where they persistently think of everything in terms of length only (e.g., they think a skinny triangle "takes up more room" than a fat one). Try to write a definition of "area" that will work even on such a student. Also be sure to distinguish area from volume. (b) How could you compare two triangles such as those at right to determine which is bigger?

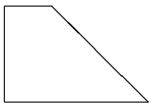


J4 *Mini-case study 1.* Pose, to a group of K–8 students, a question from one of the cases we have read or seen, or from one of our class discussions. (Make sure it is appropriate to the students' grade level.) Write about your question, what you expected, and what actually happened. Did anything surprise you? Please describe specific examples of what your students say and do. Examining the work of a few students in detail may be more helpful than trying to incorporate the responses of every student. Think of this as a dry run for the case study assignment.

J5 *Mixing units.* (a) Consider a rectangle whose sides measure 6 inches by 1 foot. What is its area? (in square inches? in square feet? in other units?) What does it mean to say that *Area = Length* $\times$ *Width*?

(b) Sand is a level 6 inches deep in a sandbox that measures 4 feet by 2 yards. What is the volume of sand in the sandbox? (c) What implications or significant ideas regarding measurement units are entailed in the two problems above?

J5.5 (don't turn in) Here is another "Crazy Cake" like those in MSP2. In this trapezoid, one base is three times as long as the other. Show how you can divide the shape into two equal shares. How can you prove the areas are equal?



J6 *Changing Linear Dimensions II.* (a) Find some cubes and build a rectangular solid. Now build a second solid that doubles all the edges of the first. What happens to the area of each face? What happens to the overall surface area of the solid? What happens to the volume? What do you think would happen to each of these quantities if you instead tripled the length of each edge? (b) Conjecture by what factor the length of each edge would have to increase in order for the volume of the original cube to double. (c) What important ideas do you see being brought up in these questions? What questions do you have about them?

J7 *One-problem paper.* As a dry run for [the first part of] the two-problem paper, write up a derivation for the area of a circle, following the relevant prompt in the coursepack for Session 2. See the guidelines for the two-problem paper.

J8 *Curriculum evaluation.* You will conduct an evaluation of the ideas entailed in a single lesson from a set of mathematics curriculum materials. See DMI handout for full prompt.

J9 *Mini-case study 2.* Pose to your students a question involving one of the topics we have discussed recently in class. Follow the same guidelines as in the first mini-case study.

J10 *Synthesis.* Reflecting over your thoughts and experiences during the course, write one short paragraph on each of the following (for a total of four): (a) changes or developments in key ideas, relative to those you identified in J1 at the beginning of the course; (b) the most striking mathematical idea of the course so far; (c) the most striking idea of the course so far about student learning/thinking; (d) a mathematical idea that still puzzles or intrigues you.

2. Student interview

In order to develop (or strengthen) the habit of attending to student thinking in detail, you will conduct an interview with a student from your class to assess her/his understanding of a specific mathematical topic. You may choose the student and topic, but the interview should involve a topic related to measurement (and not mere computation). Begin by obtaining all necessary permissions to conduct and record (audio or video) the interview; explain to all interested parties (including the student!) that you need the student's help for a class in which you are studying how students learn, and this interview will not affect her/his grades; it will just help you understand how (s)he thinks. (Recording the interview will keep you from needing to make detailed notes during it.)

Before the interview, get a copy of recent written work by the student showing her/his ability to reason and problem-solve (the work need not be error-free, but there should be enough progress made to discuss the problem). Make sure the student is familiar with the paper, and begin the interview by asking him/her to explain the work, including what difficulties s/he encountered.

Continue the interview by asking further questions about the mathematical topic involved (see the handout on interviewing tips on the course web site). You will need to use both pre-prepared questions and ad hoc follow-up questions to develop a coherent line of questioning. Remember that *in order to determine the limits of a student's knowledge, you must continue until you reach a question which the student either cannot answer or answers incorrectly for reasons other than a simple careless error*. You should be able to do this without making the student feel badly.

After the interview, use your recording to make a more detailed analysis of the student's thinking, with regard to both problem-solving abilities and knowledge of the particular mathematical topic. Begin with a brief introduction to provide context. Give an overall narration of the interview (e.g., say what specific tasks or problems you asked the student to work on). Use specific details or quotes to support your analysis. Conclude your write-up with an explicit summary of what the student knows, what the student does not know, and what the student is ready (or needs) to work on next (see interview tips handout for more).

3. Case study

During the course we will read and discuss in class several case studies, all describing events in other teachers' classrooms. For this assignment, you are to write a short (roughly 3–5 pages) case study describing a mathematical discussion involving one or more students, similar to these cases. A case is neither a complete transcript of a lesson nor as prefabricated as an interview, although it is very helpful to include direct quotes and dialogue from students.

You must base your case on a conversation for which you were present, and preferably in which you were involved, but it could come out of a lesson you observed, or a conversation among two or more students. You may choose to narrow in on one or two students, or on one small group, or you may describe a whole-class conversation. The most important thing is that the episode illustrate some aspect of children's mathematical thinking, as well as your reflection on larger teaching issues it raises for you. It must also center on a mathematical topic involving measurement.

In writing your case study, begin by describing briefly the class's larger context (including grade level) and the mathematical topic; then report the relevant parts of the conversation in as much detail as you can manage. Include your thoughts as you work with the students. Finish by summarizing your evaluation of the students' thinking and identify what issues and questions the case raises for you. *It is important that your reflection address teaching issues beyond the one topic and set of students involved*, in order to document your ability as a reflective practitioner to make connections that inform your teaching practice more broadly.

We will discuss the writing of cases in more detail before they are due, but you are encouraged to begin sooner if you have a good conversation fresh in your mind. I will be glad to help you.

4. 2-problem paper

In order to understand the concepts underlying measurement (including teaching it), you must gain experience in explaining its applications. As a summative evaluation of the mathematical portion of this course, you will submit a paper detailing your mathematical work on a *college-level* problem from this course which you solved completely, and a problem from K–8 mathematics which you believe is related. You *must* check the course web page or meet with me individually to approve and verify the problem you wish to write up.

For the college-level problem, give a thorough explanation of the original problem (paraphrased), its context, the strategies you used to approach it, what the solution is (and why! that's the tricky part), and what the solution means in context. Distinguish carefully between conjectures and rigorous arguments. Feel free to use drawings, graphs, diagrams, tables, etc. if necessary. Cite outside sources clearly.

Also select a single problem (*not* a lesson) from K–8 mathematics (possibly multi-part, and preferably from your own classroom) which you believe entails measurement concepts similar to those involved in the college-level problem, and explain in detail what those concepts are, clarifying in a paragraph what common ideas the two problems share. Include the prompt in your paper. Limit your analysis to properties of the problems themselves; do not focus on student work (unlike in the other major papers).

I strongly encourage you to show me a draft of your paper before final submission.

5. Lesson paper

In this course we will study the teaching and learning of ideas related to measurement in K–8 mathematics. As a summative evaluation of the pedagogical aspects of this course, you will develop or select an exemplary lesson which fosters learning these concepts, teach and document the lesson, and give a short (10-minute) presentation to the class on it. The lesson draft checkpoint includes items 1–3 below. The final lesson paper you submit must include *all* of the following components:

1. Select or develop a problem intended for use with the students you teach, which involves a major idea from this course. You may use or adapt a problem from class materials, but be sure it is appropriate for the target audience. (Say where you got it from, and, if you have used it before, in what capacity, and what you learned from it.) The best lessons tend either to integrate multiple strands of mathematics to illustrate connections, or to address significant conceptual issues within a single strand as a summative activity following multiple experiences in developing and exploring a concept. Specify prerequisite knowledge.
2. Write a paragraph explaining what concepts from this course are entailed in this problem. (You may use deconstruction if it helps you identify them, but write in paragraph form.)
3. Add to the above written descriptions a short sketch of how you plan to use the problem in a lesson, and meet with your instructor to discuss your progress. (This is the lesson draft checkpoint. The above items will also form part of your final paper.)
4. Write a lesson plan that uses the problem/activity as a significant problem-solving opportunity with your students. Include closure activities, and important discussion points.
5. Teach the lesson to your students (see me if this is problematic). Then write a one-page reflection on how the lesson went, including what strategies students used to approach the problem, what ideas were raised in its discussion, and to what extent your students' understanding of the underlying measurement concepts—or ability to apply them—changed as a result of the lesson. Be specific.
6. Make a one-page handout (you may use front and back if necessary, but it *must* fit on one sheet) summarizing your lesson for the class. Include the problem, grade level, mathematical topics addressed, and anything your colleagues would need to know in order to use the lesson, including (briefly) any difficulties the students tended to encounter. The handout should *not* be the same as your lesson plan (select details!), and must be turned in with the main paper.
7. Give a brief (10-minute) presentation to the class on this lesson, using the handout, at our last class meeting.

I encourage you to discuss this project with me as often as you like, throughout the semester. A preliminary draft of the selected problem and lesson idea (not [necessarily] yet taught) is due at Session 7 (see step 3 above). Final documentation is due at Session 11, including a handout, with the presentations to be given at Session 16.

Bibliography

Michael T. Battista, How many blocks? *Mathematics Teaching in the Middle School* 3(6): 404–411, March–April 1998.

Elana Joram, Benchmarks as tools for developing measurement sense, in Douglas H. Clements and George Bright (eds.), *Learning and teaching measurement*, 2003 NCTM Yearbook. Reston, VA: NCTM, 2003. pp. 57–67.

Constance Kamii and Kathy Long, The measurement of time: transitivity, unit iteration, and the conservation of speed, in Douglas H. Clements and George Bright (eds.), *Learning and teaching measurement*, 2003 NCTM Yearbook. Reston, VA: NCTM, 2003. pp. 168–179.

Paula Maida and Michael Maida, How does your doughnut measure up?, *Mathematics Teaching in the Middle School* 11(5): 212–219, Dec. 2005/Jan. 2006.

Deborah Schifter, Virginia Bastable, and Susan Jo Russell, *Measuring Space in One, Two, and Three Dimensions Casebook*. Parsippany, NJ: Dale Seymour/Pearson, 2002.

Sandra Davis Trowell and Anne M. Reynolds, How long is its projection?, *Mathematics Teaching in the Middle School* 9(8): 444–448, April 2004.

Calendar

A tentative schedule with topics is given below (subject to updating).

Sess.	Date	Topic	Readings/Cases Due	Assignments Due
1	5/12	Introduction to measurement	—	J1 key ideas
2	5/17	Students' ideas about meas. size (MSP1)	MSP1	J2 benchmarks*
3	5/19	Standard and nonstandard units	Joram, 4 cases	J3 area refl., addl.
4	5/24	Measuring length (MSP3)	MSP3	Interview*
5	5/26	Composing & decomposing space (MSP2)	MSP2	J4 mini-case*, addl.
6	5/31	Meas. areas of rect. & triangles (MSP4)	MSP C18,19,20,22,23	J5 mixing units
7	6/02	Measuring composite areas (MSP5)	MSP C24	Case study*, [J5.5], addl.
8	6/07	Measuring irregular areas	MSP C21	Lesson draft
9	6/08	Measuring volume I (MSP6)	MSP6, MSP C5, Battista	J6 Chg Lin Dim II
10	6/09	Measuring volume II; algebra of units	Maida & Maida	J7 1PP, addl.
11	6/14	Boundary vs. interior I (MSP7)	MSP7	Lesson paper*
12	6/15	Boundary vs. interior II: optimization	—	J8 curr. eval.
13	6/16	Conclusions on measuring size (MSP8)	MSP8	2-problem paper
14	6/21	Measuring time	Kamii & Long, addl.	J9 mini-case*
15	6/22	"Heavy": mass and related quantities	Trowell & Reynolds, addl.	J10 synthesis, [rewrite]
16	6/23	Final presentations	—	Give presentations

addl. means to see the reader or coursepack for further details.
MSPn means Chapter n of the DMI casebook. MSP Cn means Case n only.
See bibliography for further details of readings (nonelectronic readings are available at the UTA Libraries).

Note on study time: Summer courses take a sixteen-week semester and compress it into five weeks. That's a compression factor of more than three! Not only does this time compression leave students less time to "unpack" and reflect between class meetings, but it also means that in order to engage fully in the course, one has to spend more than five times as many hours per day outside of class on coursework than would be true during a long semester. In particular, instead of the usual rule of thumb of six hours per week outside of class for every three hours per week spent in class, the proportion becomes 18 hours per week out of class for 9 hours per week in class. This is equivalent to a half-time job. Please be careful to plan accordingly.

University Policies

Attendance: At The University of Texas at Arlington, taking attendance is not required. Rather, each faculty member is free to develop his or her own methods of evaluating students' academic performance, which includes establishing course-specific policies on attendance.

Drop Policy: Students may drop or swap (adding and dropping a class concurrently) classes through self-service in MyMav from the beginning of the registration period through the late registration period. After the late registration period, students must see their academic advisor to drop a class or withdraw. Undeclared students must see an advisor in the University Advising Center. Drops can continue through a point two-thirds of the way through the term or session. It is the student's responsibility to officially withdraw if they do not plan to attend after registering. **Students will not be automatically dropped for non-attendance**. Repayment of certain types of financial aid administered through the University may be required as the result of dropping classes or withdrawing. For more information, contact the Office of Financial Aid and Scholarships (http://wweb.uta.edu/ses/fao).

Americans with Disabilities Act: UT Arlington is on record as being committed to both the spirit and letter of all federal equal opportunity legislation, including the *Americans with Disabilities Act (ADA)*. All instructors at UT Arlington are required by law to provide "reasonable accommodations" to students with disabilities, so as not to discriminate on the basis of that disability. Any student requiring an accommodation for this course must provide the instructor with official documentation in the form of a letter certified by the Office for Students with Disabilities, University Hall 102. Only those students who have officially documented a need for an accommodation will have their request honored. Information regarding diagnostic criteria and policies for obtaining disability-based academic accommodations can be found at www.uta.edu/disability or by calling the Office for Students with Disabilities at (817)272-3364.

Title IX: The University of Texas at Arlington does not discriminate on the basis of race, color, national origin, religion, age, gender, sexual orientation, disabilities, genetic information, and/or veteran status in its educational programs or activities it operates. For more information, visit uta.edu/eos. For information regarding Title IX, visit www.uta.edu/titleIX.

Academic Integrity: Students enrolled in this course are expected to adhere to the UT Arlington Honor Code:

> *I pledge, on my honor, to uphold UT Arlington's tradition of academic integrity, a tradition that values hard work and honest effort in the pursuit of academic excellence. I promise that I will submit only work that I personally create or contribute to group collaborations, and I will appropriately reference any work from other sources. I will follow the highest standards of integrity and uphold the spirit of the Honor Code.*

UT Arlington faculty members may employ the Honor Code as they see fit in their courses, including (but not limited to) having students acknowledge the honor code as part of an examination or requiring students to incorporate the honor code into any work submitted. Per UT System *Regents' Rule* 50101, §2.2, suspected violations of university's standards for academic integrity (including the Honor Code) will be referred to the Office of Student Conduct. Violators will be disciplined in accordance with University policy, which may result in the student's suspension or expulsion from the University. *Papers involving plagiarism will receive an indelible grade of zero.*

Electronic Communication: UT Arlington has adopted MavMail as its official means to communicate with students about important deadlines and events, as well as to transact university-related business regarding financial aid, tuition, grades, graduation, etc. All students are assigned a MavMail account and are responsible for checking the inbox regularly. There is no additional charge to students for using this account, which remains active even after graduation. Information about activating and using MavMail is available at http://www.uta.edu/oit/cs/email/mavmail.php.

To obtain your NetID or for logon assistance, visit https://webapps.uta.edu/oit/selfservice/. If you are unable to resolve your issue from the Self-Service website, contact the Helpdesk at helpdesk@uta.edu or (817)272-2208.

Student Feedback Survey: At the end of each term, students enrolled in classes categorized as lecture, seminar, or laboratory will be asked to complete an online Student Feedback Survey (SFS) about the course and how it was taught. Instructions on how to access the SFS system will be sent directly to students through MavMail approximately 10 days before the end of the term. UT Arlington's effort to solicit, gather, tabulate, and publish student feedback data is required by state law; students are strongly urged to participate. www.uta.edu/sfs

Final Review Week: A period of five class days prior to the first day of final examinations in the long sessions shall be designated as Final Review Week. The purpose of this week is to allow students sufficient time to prepare for final examinations. During this week, there shall be no scheduled activities such as required field trips or performances; and no instructor shall assign any themes, research problems or exercises of similar scope that have a completion date during or following this week *unless specified in the class syllabus*. During Final Review Week, an instructor shall not give any examinations constituting 10% or more of the final grade, except makeup tests and laboratory examinations. In addition, no instructor shall give any portion of the final examination during Final Review Week. During this week, classes are held as scheduled. In addition, instructors are not required to limit content to topics that have been previously covered; they may introduce new concepts as appropriate.

Grade Grievances: Any appeal of a grade in this course must follow the procedures and deadlines for grade-related grievances as published in the current graduate catalog.

Emergency Exit Procedures: Should we experience an emergency event that requires us to vacate the building, students should exit the room and move toward the nearest exit. When exiting the building during an emergency, one should never take an elevator but should use the stairwells. Faculty members and instructional staff will assist students in selecting the safest route for evacuation and make arrangements to assist individuals with disabilities.

Student Support Services: UTA provides a variety of resources and programs designed to help students develop academic skills, deal with personal situations, and better understand concepts and information related to their courses. Resources include tutoring, major-based learning centers, developmental education, advising and mentoring, personal counseling, and federally funded programs. For individualized referrals, students may visit the reception desk at University College (Ransom Hall), call the Maverick Resource Hotline at 817-272-6107, send a message to resources@uta.edu, or view the information at www.uta.edu/resources.

Emergency Phone Numbers: In case of an on-campus emergency, call the UTA Police Department at **817-272-3003**, or dial 911.