Math 1330–004 Sample Final Exam 1, Fall 2008

INSTRUCTIONS: You have 2 1/2 hours to work the following problems. You may use your class notes and papers and a calculator, but may not work with others. Try to spread your time evenly; do not spend too long on any one problem. If you have questions, raise your hand. Good luck! NOTE: You will not be allowed to leave the room and return, once the exam has started.

1. (a) Write the fraction 1/35 as a repeating decimal. Explain how you know it repeats.
   (b) Write the repeating decimal $7.\overline{07}$ as an "improper" fraction (i.e., numerator>denominator).

2. (a) Write the difference of the repeating decimals 0.121212... − 0.090090090... as a (single) common fraction.
   (b) A student asked to perform the above subtraction protests that these two numbers cannot be subtracted "because you have to start at the right, and you can't ever get all the way to the right, because they keep on going." How do you respond?

3. (a) Write a short "story problem" that models the addition $2\frac{1}{2} + \frac{3}{4}$.
   (b) Which of the three meanings of fraction identified in class was used in your problem? Which of the three is most common in general?

4. (a) Find all numbers between 1 and 150 which have exactly 10 factors.
   (b) Among the numbers from 1 to 1000, are there more numbers with exactly 2 factors or exactly 3 factors? Explain.

5. Describe (in terms of prime factorizations) the numbers which have exactly (a) 77 factors, (b) 57 factors.

6. (a) Perform the multiplication $35_{twelve} \times 24_{twelve}$ in base twelve; show all your work, *in base twelve*, including partial products. Note this problem is **not** thirty-five times twenty-four — the numbers are given in base twelve.

   (b) Divide 10011.1_{two} by 11_{two} (in base two). Show your work.

7. Find two stamps with which you can make any amount \$1 or more, but which are as large as possible otherwise.

8. (a) As we discussed in class, it is only necessary to look at the last digit of a number in order to determine whether it is divisible by which other numbers in base ten?
   (b) Why is this (previous statement) true, in place value terms?
   (c) Therefore, if we were instead operating in a base *six* number system, it would only be necessary to look at the last digit of a number when testing divisibility by which numbers?
   (d) If we write a number in base *seven*, by looking at its last digit in this representation we can determine divisibility by which numbers?

9. Deconstruct the following problem into its essential elements (list at least four), taking into account the effect each may have upon the ways the problem can be solved.

   > Nineteen children are going to the circus. Five children can ride in each car. How many cars will be needed to take all the children to the circus?

10. An important problem-solving ability is being able to solve a problem in more than one way. Mention a problem you have solved in this course in more than one way, and describe the techniques you used on it. (Do not re-solve the problem, just describe the methods.)

Math 1330–004 Sample Final Exam 2, Fall 2008

INSTRUCTIONS: You have 2 1/2 hours to work the following problems. You may use your class notes and papers and a calculator, but may not work with others. Try to spread your time evenly; do not spend too long on any one problem. Show your work. If you have questions, raise your hand. Good luck! NOTE: You will not be allowed to leave the room and return, once the exam has started.

1. (a) Find the product of these base two numbers. Give your answer in base two.

$$\begin{array}{rr} & 101_{two} \\ \times & 110_{two} \\ \hline \end{array}$$

   (b) Divide forty-five by six in base four; express any remainder in heximal (i.e., continue the division past the ones place until you have no remainder left). Begin by converting to base four. Show your work.

2. List all the fractions whose decimal representation terminates no more than two places past the decimal point, and whose numerator is 1.

3. How many factors does the number 4400 have? (Do *not* attempt to list them all.)

4. Find the first number greater than 100 which has exactly 9 factors.

5. If you have an unlimited supply of 4-cent and 7-cent stamps, which of the following amounts can you make? Why? (a) 13 cents (b) 18 cents (c) $2.01

6. In which bases does the place value (decimal) representation for the fraction 3/4 terminate? Why?

7. List 6 different problem solving techniques you have used on problems this semester. A technique is a general approach which applies across a broad range of problems.

8. (a) In which base less than twenty do the three basic divisibility tests cover divisibility by all but two of the numbers 2–10?

   (b) In which bases do the three basic divisibility tests cover divisibility by all one-digit (in that base) numbers?

9. Give five representations for the fraction 5/6 which are as substantively different as possible.

10. Deconstruct the following problem into its essential elements (list at least four), taking into account the effect each may have upon the ways the problem can be solved.

    > You have $1\frac{1}{2}$ oranges. If this is enough to make 3/5 of an adult serving, how many oranges constitute 1 adult serving?

As you can see from the sample exams, topics include multiplication and division in other bases, representing fractions, converting between fractions and decimals, deconstructions, divisibility tests, applications of the stamps problem, factors, and problem solving.

Also see problems in the coursepack on pp. 46, 54, 66, 68, 69, 71, 75.